Find the zeros of each function analytically (No calc. zero funct. or graphs/tables)

(a) $f(x) = 6x^3 + 2x^2 - 20x - 16$

$2(3x^3 + x^2 - 10x - 8)$

(b) $f(x) = x^3 + 3x + 1$

No rational zeros

Write equation for the tangent line to f(x) through the indicated point

(a) $y = \frac{1}{x-1}$ at $x = 2$

(b) $f(x) = \sqrt{1+x}$ at $x=0$

$\frac{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16}{\pm 1, \pm 2, \pm 3, \pm 6}$

$\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3}$

$$\begin{array}{r|rrrr} -1 & 6 & 2 & -20 & -16 \\ & & -6 & 4 & 16 \\ \hline & 6 & -4 & -16 & 0 \end{array}$$

$(x+1)(6x^2 - 4x - 16)$

$x = -1, 2, -\frac{4}{3}$

$y' = -1(x-1)^{-2}$

$y' = -1(2-1)^{-2}$

$y' = -1$ slope at $x=2$

$y = m(x - x_1) + y_1$

$$\boxed{y = -1(x-2) + 1}$$

(b) $y = \sqrt{1+x}$, $x = 0$

$y' = \frac{1}{2}(1+x)^{-\frac{1}{2}}$

$y = \frac{1}{2}$ slope at $x=0$

$y = \frac{1}{2}(x - 0) + 1$

$$\boxed{y = \frac{1}{2}x + 1}$$

Linearization

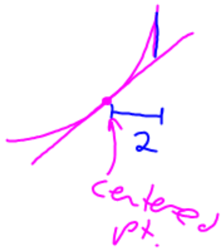
→ we can think of any differentiable function on a very small interval (pt.) as linear.

→ tangent line → $L(x)$ linearization of the funct. at x

→ $L(x) = f(a) + f'(a)(x-a)$

$y = y_1 + m(x - x_1)$ with $m = f'(a)$, pt $(a, f(a))$

Accuracy



$$\left| f(a+2) - L(a+2) \right|$$

Example of finding accuracy

Estimate $\sqrt{123} \rightarrow \sqrt{x}$ $\qquad f'(x) = \frac{1}{2}(x)^{-\frac{1}{2}}$

$L(x) = f(a) + f'(a)(x-a)$ (tangent line)

$= f(121) + f'(121)(x-121)$

$= 11 + \frac{1}{2}(121)^{-\frac{1}{2}}(123-121)$

$L(x) = 11 + \underbrace{\frac{1}{22}}_{\frac{1}{2}\cdot\frac{1}{\sqrt{121}}}(123-121)$

$L(x) = 11\frac{1}{11}$

$L(x) \approx 11.\overline{09}$

Actual $\sqrt{123} = 11.09053651$

11.09090909

Error Less than 10^{-3}

Estimate, by using Linearization, $\sqrt[3]{123}$

→ tangent line to $\sqrt[3]{x}$ at $x = 125$

$$f'(x) = \frac{1}{3}(x)^{-2/3}$$

$$f'(125) = \frac{1}{3}(125)^{-2/3}$$

$$\frac{1}{3} \cdot \frac{1}{25} = \frac{1}{75} = m$$

$$y = m(x - x_1) + y_1$$

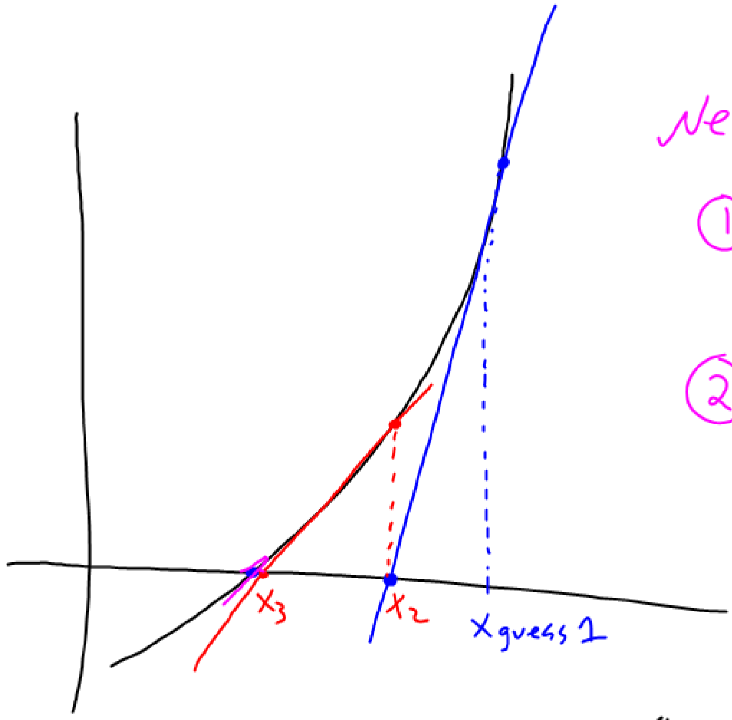
$$L(x) = \frac{1}{75}(\underline{\underline{x}} - 125) + 5$$

$$L(123) = \frac{1}{75}(123 - 125) + 5$$

$$L(123) \approx 4.97\overline{33}$$

$$\text{Actual} \approx 4.973189\ldots$$

Error is less than 10^{-3}



Newton's Method

① Guess a zero, try to get close

② $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_2 \longrightarrow$ guess
$x_3 \longrightarrow x_2$
$x_4 \longrightarrow x_3$

On Calc.

Y_1 = Funct.

Y_2 = NDER of Funct.

main screen

guess $\rightarrow x$

$x - \frac{Y_1}{Y_2} \rightarrow x$

E

$\boxed{4.5}$

Q.R. 1, 2, 4, 7–10

4.5 #1, 2, 3, 7, 8a, 9, 11, 12, 15–17

Read 4.5 Differentials